OPTIMAL POSITION ANALYSIS OF SECONDARY SYSTEMS IN YIELDING STRUCTURES

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ABSTRACT: Based on the code for seismic design of building in China, primary structures usually yield due to severe earthquake loading in practice, so non-linearization of primary structures plays an important role in the dynamic response analysis. In this article, random response of secondary systems mounted on nonlinear primary system under random excitation is studied by using equivalent linearization method, and a 10-storey shearing structures is selected as an example. Using mean square response of secondary system as the objective function the optimal position and corresponding optimal parameters are obtained, and some valuable conclusions are proposed.

KEYWORDS: secondary systems, non-linearization, mean square response, equivalent linearization, optimal position.

INTRODUCTION

Primary-secondary systems are usually composed of two parts: primary system and secondary system. The equipment, furniture, architectural elements, and other non-structural components attached to the floors and walls of buildings are called secondary system, and structures which they are mounted on are called primary systems. In the earthquake, although structures maintain serviceable, the failure of secondary systems would also bring severe consequence. Reconnaissance reports and surveys on the seismic performance of non-structural components during past earthquakes report that failure of secondary systems constitutes a major portion of economic losses. In critical facilities, the direct and indirect losses can be more than the cost of replacing the collapsed building or structures.

Much research work has been done on the dynamic behaviour of secondary systems connected to a primary structure during the last three decades [1-6]. As a result of an effort from the engineering profession to ensure the survivability of critical installations, such as piping systems and control panels, in nuclear power plants, some simple progress has been generated. However, the simple methods, which have been developed for the design of these components, are approximate methods that just are suitable for particular situations. There are mainly three approaches as follows: floor response spectra, dynamic response analysis of combined systems or cascade systems, and approximate method based on perturbation technique. The composite characteristics of such systems don't match with usual structures, and traditional methods used for the analysis of classically damped structural systems, are not applicable. Although analysis of such structural systems without considering interaction between the primary and the secondary structural systems, is easy and economical, this analysis gives incorrect responses of the secondary system when the secondary system is not very light as compared to the supporting primary system, and when the frequency of vibration of the secondary system matches with one of the dominant frequencies of the supporting structural system. In such situations, responses of the secondary system, calculated by considering the interaction between the primary-secondary system give more realistic responses of the secondary system.

The earthquake is a complex and random process, which can not be described in determinate time history. In the study, random input is adopted as the excitation, and seismic response of primary secondary system is investigated by the theory of random. Based on the code for seismic design of building in China, primary structures usually yield due to severe earthquake loading in practice, so non-linearization of primary structures plays an important role in the dynamic response analysis. Most of the published papers are concerned with elastic primary–secondary structures and only in a relatively small amount of work inelastic structural behavior is included [7-9]. In this article, random response of secondary systems mounted on nonlinear primary system under random excitation is studied by using equivalent linearization method, in which the mean square response is selected as objective function for optimization design of secondary systems.

BASIC EQUATIONS

The primary-secondary system in the research is made up of shearing structures and accessory secondary systems, as is described in the following figure. Optimal position, which usually means the minimum seismic response of secondary system, is studied on the basis of random theory, and primary structure is subjected to the random excitation of White Noise. In the paper, structures are considered to yield due to severe earthquake excitation and Bouc-Wen hysteretic model is adopted to describe the nonlinearity of primary system. In the last three decades, several methods are proposed in order to obtain the appropriate response value, and the monte-carlo method is considered as the exact solution, which requires a mount of calculation and is difficult for actual application. Besides, some approximate methods are also proposed, representative one of which is the generalized equivalent linearization method. Generalized equivalent linearization method is firstly introduced by Caughey in 1963, then is improved separately by Bobori, Kaul, Penzien and Y. K. Wen, and it can solve both the heavy and light nonlinearity problems.

The Fig.1 is the schematic diagram of the model of primary-secondary systems.

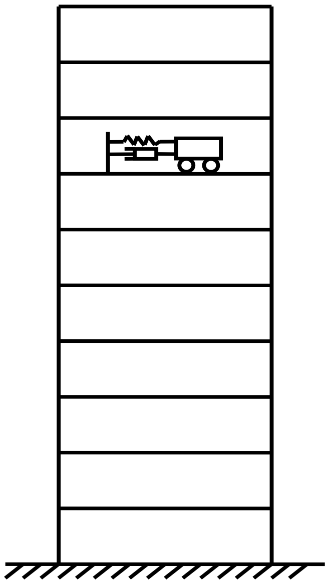


Figure.1 Schematic diagram of the model of primary-secondary

Bouc-Wen Hysteretic Model

A nonlinear model of restoring force and damping is introduced by Y. K. Wen, which describes the nonlinear restoring force in a smooth curve. In the model of restoring force, by assuming linear damping force, hysteretic restoring force can be expressed as follows:

$$g(x,\dot{x}) = c\dot{x} + \alpha kx + (1-\alpha)kz \tag{1}$$

where k and c denote the stiffness and damping coefficient of the structure, respectively; α is the ratio of the postyield to the preyield stiffness of the structure.

Using Bouc-Wen model, the hysteretic component of the restoring force of the primary strucuture is $(1-\alpha)kz$, in which z is governed by the nolinear differential equation:

$$\dot{z} = A\dot{x} - \gamma|\dot{x}||z|^{n-1}z - \beta\dot{x}|z|^n \tag{2}$$

where n, A, β, and γ are the parameters that control the shape of the hysteretic loop. Equation. (2) is capable of representing a wide class of hysteretic behavior. For simplification, a combination of the paramter values is given by $A = n = 1$, $\beta = 0.05$, and $\gamma = 0.95$ in the paper.

Generalized equivalent linearization method

Bouc-Wen model is simply defined as follows:

$$\dot{z} = A\dot{x} - r|\dot{x}|z - \beta\dot{x}|z| \tag{3}$$

and we can rewrite it in another form: $h(x,\dot{x},z) = \dot{z} - A\dot{x} + r|\dot{x}|z + \beta\dot{x}|z|$.

Based on equivalent linearization, nonlinear differential equation (3) can be replaced by following linear one:

$$\dot{z} = c_e\dot{x} + k_e z \tag{4}$$

According to the minimum of difference value between equations (3) and (4), c_e and k_e can be defined:

$$c_e = E[\frac{\partial h}{\partial \dot{x}}] = A - \gamma E(z\frac{\partial|\dot{x}|}{\partial \dot{x}}) - \beta E[|z|] = A - \sqrt{\frac{2}{\pi}}[\gamma\frac{E(\dot{x}z)}{\sigma_{\dot{x}}} + \beta\sigma_z] \tag{5}$$

$$k_e = E[\frac{\partial h}{\partial z}] = -\gamma E[|\dot{x}|] - \beta E(\dot{x}\frac{\partial|z|}{\partial z}) = -\sqrt{\frac{2}{\pi}}[\gamma\sigma_{\dot{x}} + \beta\frac{E(\dot{x}z)}{\sigma_z}] \tag{6}$$

where the values of c_e and k_e depend on the moment of x, $\dot{x}$, z and $\dot{z}$.

Due to the moment of x, $\dot{x}$, z and $\dot{z}$ is calculated on the basis of confirmation of c_e and k_e, it is a iteration process for determining the value of c_e and k_e. The iteration process can be described as follows:

(i) Assume the initial values of c_e and k_e.

(ii) Computer the moment of x, $\dot{x}$, z and $\dot{z}$ from the motion equation of system.

(iii) Obtain the new equivalent values of c_e and k_e from equation (5) and (6).

(iv) Computer the new moment corresponding to the new parameter values of c_e and k_e.

(v) Perform the termination test for constringent solution.

Repeat from step (ii) to step (v) until a convergent solution is obtained.

Equivalent linear equations of motion and objective function

Assuming secondary system is mounted on the a th floor of primary structure, and the equations of motion of primary-secondary system can be written as

$$\ddot{x}_i - \frac{f_{i-1} - f_i}{m_{i-1}} + \frac{f_i - f_{i+1}}{m_i} = 0, i = 2,3,\ldots,n, i \neq a, a+1 \tag{7a}$$

$$\ddot{x}_1 + \frac{f_1 - f_2}{m_1} = -\ddot{x}_g, i = 1, i \neq a \tag{7a}$$

$$\ddot{x}_i - \frac{f_{i-1} - f_i}{m_{i-1}} + \frac{f_i - f_{i+1} - f_s}{m_i} = 0, i = a, a \neq 1 \tag{7c}$$

$$\ddot{x}_i + \frac{f_i - f_{i+1} - f_s}{m_i} = -\ddot{x}_g, i = a, a = 1 \tag{7d}$$

$$\ddot{x}_i - \frac{f_{i-1} - f_i - f_s}{m_{i-1}} + \frac{f_i - f_{i+1}}{m_i} = 0, i = a+1, a+1 \leq 10 \tag{7e}$$

$$\ddot{x}_s - \frac{f_a - f_{a+1} - f_s}{m_a} + \frac{f_s}{m_s} = 0 \tag{7f}$$

where $f_i = c_i\dot{x}_i + \alpha_i k_i x_i + (1-\alpha_i)k_i z_i, i = 1,\ldots,n$, $f_{n+1} = 0$; $f_s = c_s\dot{x}_s + k_s x_s$; $\dot{z}_i = c_{ei}\dot{x}_i + k_{ei}z_i$; m_i, c_i and k_i denote the mass, stiffness and damping coefficient of the i th floor of primary structure; x_i denotes the inter-storey displacement; $\ddot{x}_g$ is the horizontal ground acceleration, which is assumed as a white-noise random process with a constant spectral density of S_g.

The above equations of motion can be rewritten in the form of first order differential matrix equation of state vector.

$$\{\dot{u}\} + [F]\{u\} = -\{V\}\ddot{x}_g \tag{8}$$

where $\{u\}^T = [\{u_1\}^T, \{u_2\}^T, \ldots, \{u_n\}^T, \{u_s\}^T]; \{u_i\}^T = [x_i, \dot{x}_i, z_i], 1 \leq i \leq n; \{u_s\}^T = [x_s, \dot{x}_s]; \{V\}^T = [0,1,0,\ldots,0];$

$$[F] = \begin{bmatrix} F_{11} & F_{12} & & & & & & & \\ F_{21} & F_{22} & F_{23} & & & & & & \\ & \cdots & \cdots & \cdots & & & & & \\ & & F_{a,a-1} & F_{a,a} & F_{a,a+1} & & & & F_{a,s} \\ & & & F_{a+1,a} & F_{a+1,a+1} & F_{a+1,a+2} & & & F_{a+1,s} \\ & & & & \cdots & \cdots & \cdots & & \\ & & & & & F_{n,n-1} & F_{n,n} & & \\ & & & F_{s,a} & F_{s,a+1} & & & & F_{s,s} \end{bmatrix}; F_{11} = \begin{bmatrix} 0 & -1 & 0 \\ h_1\alpha_1 k_1 & h_1 c_1 & h_1(1-\alpha_1)k_1 \\ 0 & -c_{e1} & -k_{e1} \end{bmatrix};$$

$$F_{12} = \begin{bmatrix} 0 & 0 & 0 \\ -h_1\alpha_2 k_2 & -h_1 c_2 & -h_1(1-\alpha_2)k_2 \\ 0 & 0 & 0 \end{bmatrix}; F_{i,i-1} = \begin{bmatrix} 0 & 0 & 0 \\ -h_{i-1}\alpha_{i-1}k_{i-1} & -h_{i-1}c_{i-1} & -h_{i-1}(1-\alpha_{i-1})k_{i-1} \\ 0 & 0 & 0 \end{bmatrix};$$

$$F_{i,i} = \begin{bmatrix} 0 & -1 & 0 \\ (h_i + h_{i-1})\alpha_i k_i & (h_i + h_{i-1})c_i & (h_i + h_{i-1})(1-\alpha_i)k_i \\ 0 & -c_{ei} & -k_{ei} \end{bmatrix}; F_{i,i+1} = \begin{bmatrix} 0 & 0 & 0 \\ -h_i\alpha_{i+1}k_{i+1} & -h_i c_{i+1} & -h_i(1-\alpha_{i+1})k_{i+1} \\ 0 & 0 & 0 \end{bmatrix}$$

$$F_{a,s}=\begin{bmatrix}0 & 0\\ -h_a k_s & -h_a c_s\\ 0 & 0\end{bmatrix}; F_{a+1,s}=\begin{bmatrix}0 & 0\\ h_a k_s & h_a c_s\\ 0 & 0\end{bmatrix};\ F_{n,n-1}=\begin{bmatrix}0 & 0 & 0\\ -h_{n-1}\alpha_{n-1}k_{n-1} & -h_{n-1}c_{n-1} & -h_{n-1}(1-\alpha_{n-1})k_{n-1}\\ 0 & 0 & 0\end{bmatrix};$$

$$F_{n,n}=\begin{bmatrix}0 & -1 & 0\\ (h_{n-1}+h_n)\alpha_n k_n & (h_{n-1}+h_n)c_n & (h_{n-1}+h_n)(1-\alpha_n)k_n\\ 0 & -c_{en} & -k_{en}\end{bmatrix}; F_{s,a}=\begin{bmatrix}0 & 0 & 0\\ -h_a\alpha_a k_a & -h_a c_a & -h_a(1-\alpha_a)k_a\end{bmatrix}$$

$$F_{s,a+1}=\begin{bmatrix}0 & 0 & 0\\ h_a\alpha_{a+1}k_{a+1} & h_a c_{a+1} & h_a(1-\alpha_{a+1})k_{a+1}\end{bmatrix}; F_{s,s}=\begin{bmatrix}0 & -1\\ (h_a+h_s)k_s & (h_a+h_s)c_s\end{bmatrix}; h_i=\frac{1}{m_i}.$$

Add the expection of equation (8) multiplied by $\{u\}^T$ from the right with its transpose, and the response covariance matrix associated with equivalent linear equation is obtained which satisfies the ordinary matrix differential equation:

$$[\dot{S}(t)]+[F(t)][S(t)]+[S(t)][F(t)]^T=[P(t)] \tag{9}$$

where $[S(t)]=E[\{u(t)\}\{u(t)\}^T]$ is the response covariance matrix of primary-secondary system; $[P(t)]$ can be obtained by the formula: $[P(t)]=\{V\}\{V\}^T\times 2\pi I(t)$. While the random ground excitation is a stationary white-noise process with the intensity of S_g, a simple expression can be written:

$$[F][S]+[S][F]^T=\{V\}\{V\}^T\times 2\pi S_g \tag{10}$$

The stationary covariance matrix is the solution of the above matrix algebra equation, which depends on the value of equivalent linear parameters c_{ei} and k_{ei}.

Based on euqation (5) and (6) in the above part of the paper, equivalent linear parameters of each floor of primary structure can be written as:

$$c_{ei}=E[\frac{\partial h}{\partial \dot{x}_i}]=A_i-\gamma_i E(z_i\frac{\partial|\dot{x}_i|}{\partial \dot{x}_i})-\beta_i E[|z_i|]=A_i-\sqrt{\frac{2}{\pi}}[\gamma_i\frac{E(\dot{x}_i z_i)}{\sigma_{\dot{x}_i}}+\beta_i\sigma_{z_i}] \tag{11}$$

$$k_{ei}=E[\frac{\partial h_i}{\partial z_i}]=-\gamma_i E[|\dot{x}_i|]-\beta_i E(\dot{x}_i\frac{\partial|z_i|}{\partial z_i})=-\sqrt{\frac{2}{\pi}}[\gamma_i\sigma_{\dot{x}_i}+\beta_i\frac{E(\dot{x}_i z_i)}{\sigma_{z_i}}] \tag{12}$$

Bouc-Wenhere hysteretic behavior of each floor is assumed to be the same, so A_i, γ_i, and β_i is assumed to be 1, 0.95 and 0.05 respectively in the paper.

The iteration process can be described as follows:

(i) Assume the initial values of c_{ei} and k_{ei}, $1\le i\le n$.

(ii) Computer the covariance matrix $[S]$ from the equation (10).

(iii) Obtain the new parameter values of c_{ei} and k_{ei} from the equations (11) and (12).

(iv) Computer the new covariance matrix corresponding to the new parameter values of c_{ei} and k_{ei} from equation (10).

(v) Perform the termination test for constringent solution.

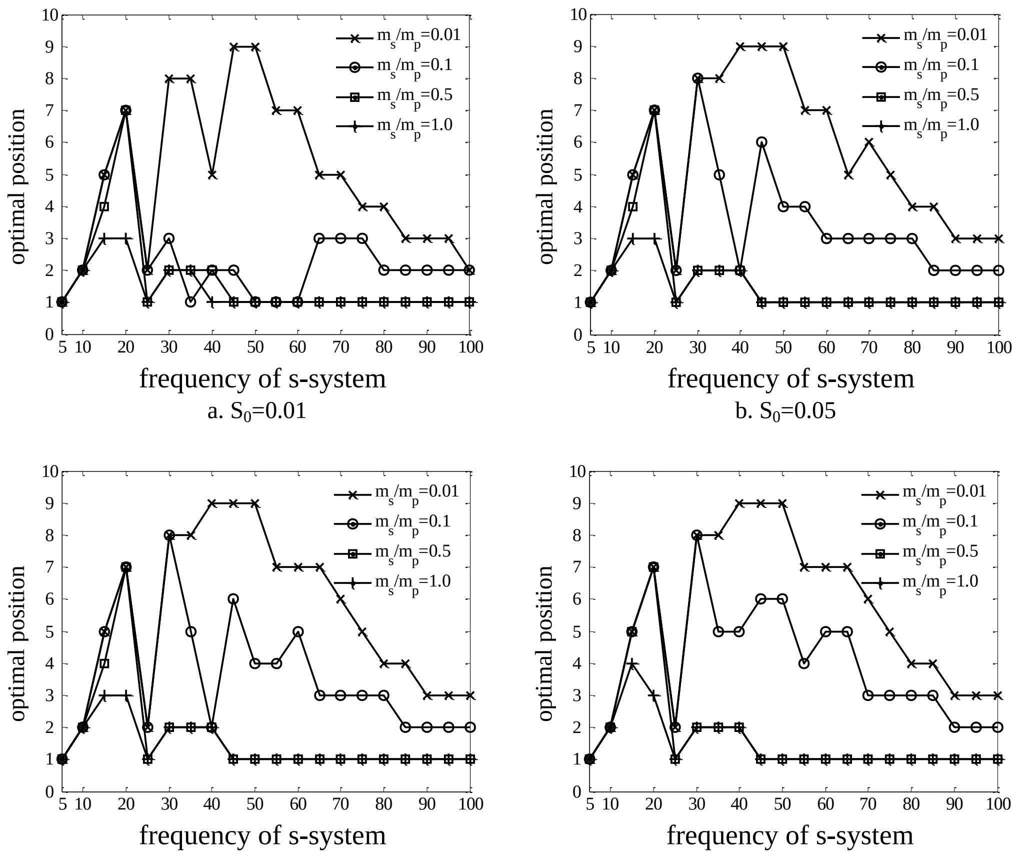
Repeat from step (ii) to step (v) until a convergent solution is obtained. According to the iterative result, the mean square response of secondary system is selected as the optimizing objective function.

NUMERICAL RESULTS AND DISCUSSIONS

In the previous research, it has been known that the factors influencing seismic response of secondary system include not only the mass, stiffness, damping ratio, and position of secondary system, but also the dynamic characteristics of primary system. Furthermore, the intensity of excitation would also have obvious effect on seismic response of secondary system while primary structures yield due to severe earthquake. Therefore, several key factors influencing optimal position of secondary system are investigated in the paper such as the mass, stiffness, damping ratio of secondary system and parameters of hysteretic model of primary structure. The crossover rate is chosen as the optimizing objective function, based on which the optimal positions with regard to different combination of key factors are computed and studied, and consequently some valuable results are obtained.

The structure model used in this analysis is a 10 multi-story building frames with a secondary system located on a certain floor. The idealized shearing structure with mass lumped on each floor is illustrated in Fig. 1. Floor masses, m_i, are $1000kg$, the inter-story stiffnesses, k_i, are $2\times10^6\ N/m$, damping ratio of structure, ξ, is 0.05. In the paper, the nonlinear model proposed by Y. K. Wen is introduced and described by using equivalent linearity method in accounting for the hysteretic restoring force characteristics.

Intensity of earthquake, Frequency and mass ratio of secondary system



a. S_0=0.01 b. S_0=0.05

c. S_0=0.1 d. S_0=0.2

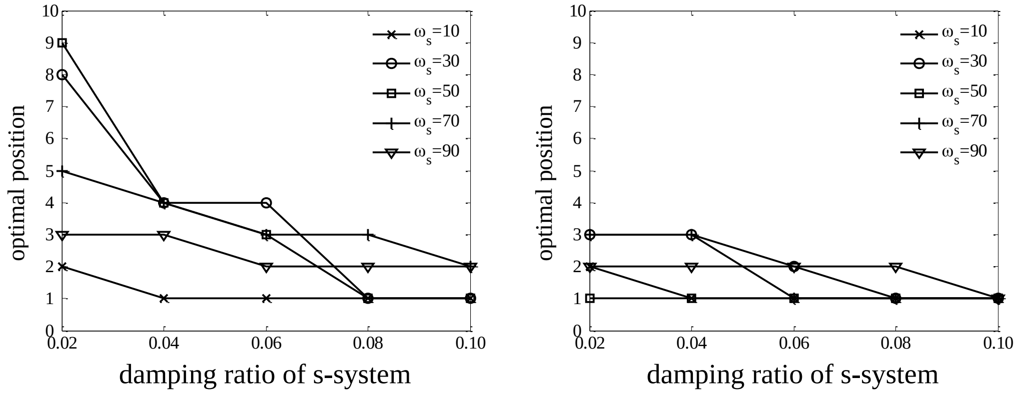
Fig.2. Effect of intensity of earthquake, frequency and mass ratio of secondary system on the optimal position

Fig.2 illustrates the effect of some key dynamic characteristics of secondary system on the optimal position, which include the intensity of earthquake, frequency and mass ratio of secondary system, etc. Form the figure, it can be observed that variations of mass and frequency of secondary system have obvious effect on optimal position while the intensity of earthquake excitation has little effect. When the mass of secondary system is small, optimal position usually locates at the bottom. In the section of low and medium order frequency, optimal position will shift form the bottom to the top with the increase of frequency of secondary system, and while the frequency of secondary system is close to primary system the optimal position usually locates at the floor corresponding to minimum location of mode. In the section of high order frequency, the effect of tuning is not obvious and optional position usually locates at the bottom.

With the increase of secondary system's mass the optimal position of secondary system shifts from the top to the bottom of primary system, and while secondary system's mass is large enough the first floor is the optimal position. The intensity of earthquake excitation has little effect on the optimal position, especially for the case that secondary system's mass is little or large enough. While secondary system's mass is medium large, the effect of intensity of earthquake is more obvious especially for the section of medium frequency, which can be seen from the curve of the $m_s/m_p = 0.1$ in the Fig.2.

Damping ratio of secondary system

Fig.3 illustrates the effect of damping ratio of secondary system on optimal position. Seismic response of the mode corresponding to secondary system's frequency would decrease due to the increased damping ratio, so the practical response is dominated by fundamental frequency. Therefore, it can be observed that with the increasing of damping ratio, optimal position shifts from the top to the bottom of primary system, and usually is not located in the bottom for the section of medium frequency of secondary system.



a. m_s/m_p=0.01, S_0=0.01　　　　b. m_s/m_p=0.1, S_0=0.01

Fig.3. Effect of damping ratio of secondary system on the optimal position

CONCLUSIONS

This paper has calculated the optimal position of secondary system mounted on nonlinear primary structure by the method of equivalent linearity, and the mean square response is adopted as the optimizing objective function. Key factors influencing the optimal position of secondary system are investigated and some valuable conclusions are gained as follows:

1: The frequency, mass and damping ratio have obvious effect on optimal position of secondary system, while the intensity of earthquake excitation has small effect on optimal position.

2: While the mass of secondary system is small, optimal position usually locates at the bottom of primary system for the low frequency section of secondary system. With the increasing of secondary system's frequency, optimal position shifts towards the top of primary system for medium frequency section, and then towards the bottom for high frequency section. Moreover, optimal position shifts towards the bottom of primary system with the increasing of secondary system's mass.

3: With the increasing of damping ratio, optimal position of secondary system shifts from the top to the bottom of primary system

ACKNOWLEDGMENTS

The authors gratefully acknowledge the financial support provided by the 111 Project (No.B08014), the Program for Changjiang Scholars and Innovative Research Team in University (No. IRT0518).

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