EJGE

A Fast Stochastic Analysis Method for Soil-Structure Interaction System

Wei Guo
State Key Laboratory of Coastal and Offshore Engineering
Dalian University of Technology, Dalian 116024, China
wei.guo.86@gmail.com

Hong-nan Li
State Key Laboratory of Coastal and Offshore Engineering
Dalian University of Technology, Dalian 116024, China
li-hong-nan@163.com

Guo-huan Liu
State Key Laboratory of Coastal and Offshore Engineering
Dalian University of Technology, Dalian 116024, China
carecivil@yahoo.com.cn

ABSTRACT

In this paper, a fast stochastic analysis method is presented to compute the seismic response of the soil-structure interaction system. Using the classical modal decomposition and the pseudo-force method, a closed form sequence is developed for iterative computation. It can account for the non-proportional damping and dynamic interaction between the soil region and the structure. Moreover, the pseudo-excitation method is introduced in the derivation for improving the computational efficiency of the stochastic analysis. The necessary and sufficient condition for convergence of the sequence is also provided. Compared with the forced decoupling method, the proposed method can significantly improve the accuracy of the results without obviously increasing computational efforts. In the end, some numerical examples are carried out to examine the accuracy and convergence of the new method.

KEYWORDS: soil-structure interaction system; stochastic analysis; iterative method.

INTRODUCTION

Soil-structure interaction is a collection of phenomena in the response of structures caused by the flexibility of the foundation soils, as well as in the response of soil region caused by the presence of structures. During the last three decades, many studies on the subject have been carried out (Seed, et al., 1975; Veletsos and Prasad, 1989; Zhang, et al., 1999; Lou and Wu, 1999; Ghiocel and Ghanem, 2002; Gao, et al., 2009). Because the soil-structure interaction system consists of two parts with distinct damping characteristics, which is usually called non-proportional damping, the equations of motion can not be represented by assembly of a series of independent oscillators. As the most common approach, the forced decoupling method (Elishakaff and Lyon, 1986) is adopted for seismic analysis of non-proportionally damped system by simply neglecting the off-diagonal elements of the transformed damping matrix, which is appealing to the design professionals because it enable the use of the traditional modal analysis methods. When the damping characteristic of the combined system is approximately identical, the error introduced by the method is so small that it can usually be ignored. However, the results obtained by this method are not exactly accurate theoretically and might introduce significant error in some cases, especially for the soil-structure interaction system, the two parts of which possess dramatically distinct properties. Therefore, in order to obtain more exact results, the complex eigenproperties via state-space approach (Foss, 1958) is developed for the modal analysis. However, the calculation of complex eigenvalues problem is cumbersome and time-consuming, and thus attempts to overcome the computational difficulties of this approach have been carried out (Lou, et al., 2003; Karen and Mohsen, 2005; Fernando and María, 2006; Hea, et al., 2007). On the other hand, based on the pseudo force method (Claret and Venancio, 1991; Lin, et al., 2003), an iterative procedure for computing the transfer function matrix of a non-classically damped system has also been developed (Jandid and Datta, 1993; Zavoni, et al., 2006). The iterative methods have more advantages than the complex modal superposition methods in terms of speed, and it retains the advantages of the real-valued modal superposition methods. Therefore, the iterative method is generally considered to be more applicable than the other methods either for time history or stochastic analysis of combined system with distinct damping characteristics.

In this paper, a fast iterative method is proposed for the stochastic analysis of soil-structure interaction system. It can account for the non-proportional damping by introducing the pseudo-force method and, at the same time, the pseudo-excitation method is adopted for improving the computational efficiency. Besides, the classical modal decomposition, which is more applicable in the practical engineering, is used in the new method instead of the complex or real mode shapes of the combined system. Finally two model of soil-structure system are taken as numerical examples to illustrate the proposed method.

FORMULATION OF ITERATIVE SEQUENCE

1. Classical modal decomposition

Figure 1 gives the general model of the soil-structure interaction system. As has been studied in the literatures (Lou and Zhao, 1994; Jin, et al., 1997; Bi and Zhang, 2003; Li, et al., 2004), the seismic equilibrium equation of the combined system can be established according to several approaches. For brevity, all of the proposed mathematic models can be expressed in the following form:

$$\boldsymbol{M}\ddot{\boldsymbol{U}} + \boldsymbol{C}\dot{\boldsymbol{U}} + \boldsymbol{K}\boldsymbol{U} = \boldsymbol{F} \tag{1}$$

in which,

$$\boldsymbol{M}=\begin{bmatrix}\boldsymbol{M}_s & 0\\ 0 & \boldsymbol{M}_g\end{bmatrix},\quad \boldsymbol{C}=\begin{bmatrix}\boldsymbol{C}_s & 0\\ 0 & \boldsymbol{C}_g\end{bmatrix}+\boldsymbol{C}_{sg},\quad \boldsymbol{K}=\begin{bmatrix}\boldsymbol{K}_s & 0\\ 0 & \boldsymbol{K}_g\end{bmatrix}+\boldsymbol{K}_{sg},\quad \boldsymbol{U}=\begin{bmatrix}\boldsymbol{U}_s\\ \boldsymbol{U}_g\end{bmatrix} \tag{2}$$

in which $\boldsymbol{M}_s$, $\boldsymbol{C}_s$ and $\boldsymbol{K}_s$ are the mass, damping and stiffness matrices of the structure respectively. $\boldsymbol{M}_g$, $\boldsymbol{C}_g$ and $\boldsymbol{K}_g$ are the mass, damping and stiffness matrices of the soil region with no structure attached on. $\boldsymbol{C}_{sg}$, $\boldsymbol{K}_{sg}$ are coupling matrices which include the damping and stiffness properties of the elements connecting the structure and the soil region. $\boldsymbol{U}$ denotes the total or relative displacement vector of the soil-structure interaction system according to the modeling method. The vector $\boldsymbol{F}$ represents the external force imposed on the soil-structure interaction system which is introduced by the ground motion.

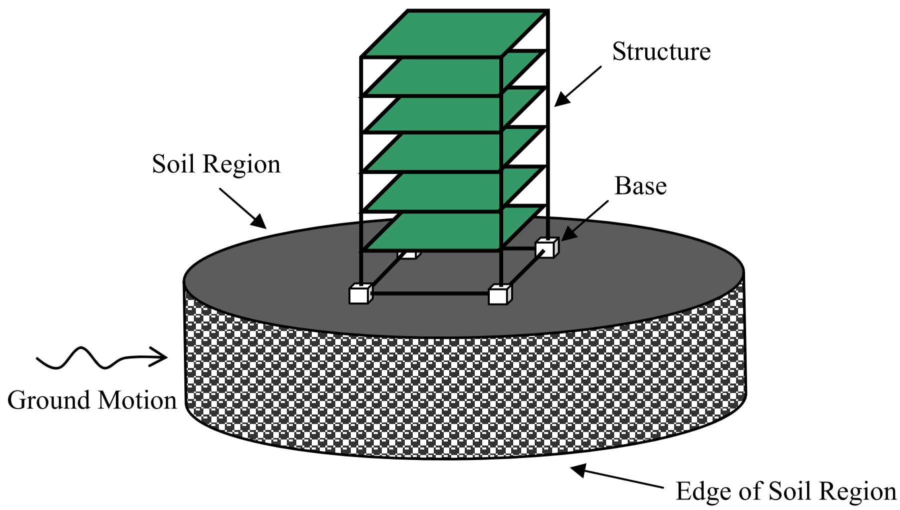

Figure 1: Schematic diagram of soil-structure interaction system

Usually modal analysis is adopted to reduce the nodal equations into a system of equations expressed in modal coordinates. However, the full eigensolution problem of the combined soil-structure system is uneconomical. A transformation of coordinates defined using concepts from the component-mode synthesis is generally adopted. It can be defined as:

$$\boldsymbol{\Phi}_s^T\boldsymbol{M}_s\boldsymbol{\Phi}_s\boldsymbol{\Omega}_s=\boldsymbol{\Phi}_s^T\boldsymbol{K}_s\boldsymbol{\Phi}_s \tag{3a}$$

$$\boldsymbol{\Phi}_g^T\boldsymbol{M}_g\boldsymbol{\Phi}_g\boldsymbol{\Omega}_g=\boldsymbol{\Phi}_g^T\boldsymbol{K}_g\boldsymbol{\Phi}_g \tag{3b}$$

in which $\boldsymbol{\Phi}_s$ and $\boldsymbol{\Phi}_g$ are generalized mode shapes corresponding to the structure and the soil region respectively. $\boldsymbol{\Omega}_s$ is the diagonal matrix listing the natural radian frequencies of the structure and $\boldsymbol{\Omega}_g$ is that of the soil region. Hence, according to the component-mode synthesis, the following coordinate transformation is obtained:

$$\boldsymbol{\Phi} = \begin{bmatrix} \boldsymbol{\Phi}_s & 0 \\ 0 & \boldsymbol{\Phi}_g \end{bmatrix} \tag{4}$$

Thus, the response $\boldsymbol{U}$ of the soil-structure system can be expressed as:

$$\boldsymbol{U} = \boldsymbol{\Phi q} \tag{5}$$

By pre-multiplying both sides of equation (1) with the matrix $\boldsymbol{\Phi}^T$, we can write the differential equations of motion as follows:

$$\boldsymbol{I}\ddot{\boldsymbol{q}} + \bar{\boldsymbol{C}}\dot{\boldsymbol{q}} + \bar{\boldsymbol{K}}\boldsymbol{q} = \bar{\boldsymbol{P}} \tag{6}$$

in which $\boldsymbol{I}$ is the identity matrix, and

$$\bar{\boldsymbol{C}} = \boldsymbol{\Phi}^T \boldsymbol{C\Phi} = \begin{bmatrix} \boldsymbol{\Phi}_s^T \boldsymbol{C}_s \boldsymbol{\Phi}_s & \\ 0 & \boldsymbol{\Phi}_g^T \boldsymbol{C}_g \boldsymbol{\Phi}_g \end{bmatrix} + \boldsymbol{\Phi}^T \boldsymbol{C}_{sg} \boldsymbol{\Phi} = \begin{bmatrix} \bar{\boldsymbol{C}}_s & 0 \\ 0 & \bar{\boldsymbol{C}}_g \end{bmatrix} + \bar{\boldsymbol{C}}_{sg} \tag{7a}$$

$$\bar{\boldsymbol{K}} = \boldsymbol{\Phi}^T \boldsymbol{K\Phi} = \begin{bmatrix} \boldsymbol{\Phi}_s^T \boldsymbol{K}_s \boldsymbol{\Phi}_s & 0 \\ 0 & \boldsymbol{\Phi}_g^T \boldsymbol{K}_g \boldsymbol{\Phi}_g \end{bmatrix} + \boldsymbol{\Phi}^T \boldsymbol{K}_{sg} \boldsymbol{\Phi} = \begin{bmatrix} \bar{\boldsymbol{K}}_s & 0 \\ 0 & \bar{\boldsymbol{K}}_g \end{bmatrix} + \bar{\boldsymbol{K}}_{sg} \tag{7b}$$

in which $\bar{\boldsymbol{C}}_s$ and $\bar{\boldsymbol{K}}_s$ are diagonal matrices denoting the damping and stiffness characteristics of the structure in the modal coordinate system. $\bar{\boldsymbol{C}}_g$ and $\bar{\boldsymbol{K}}_g$ correspond to those of the soil region in the modal coordinate system. $\bar{\boldsymbol{C}}_{sg}$ and $\bar{\boldsymbol{K}}_{sg}$ are usually full-rank matrices representing the coupling terms of the soil region and the structure. $\bar{\boldsymbol{P}} = \boldsymbol{\Phi}^T \boldsymbol{P}$ represents the transformed external force vector in the modal coordinates. It can easily be seen that the modal responses described in equation (6) is coupled. Based on the principles from pseudo-force method, an iterative sequence to evaluate the exact response of the soil-structure interaction system can be formulated. Accordingly, the damping and stiffness matrices in the modal coordinates are decomposed as:

$$\bar{\boldsymbol{C}} = \bar{\boldsymbol{C}}_d + \bar{\boldsymbol{C}}_f \tag{8a}$$

$$\bar{\boldsymbol{C}}_d = \begin{bmatrix} \bar{\boldsymbol{C}}_s & 0 \\ 0 & \bar{\boldsymbol{C}}_g \end{bmatrix} + \mathrm{diag}[\bar{\boldsymbol{C}}_{sg}], \quad \bar{\boldsymbol{C}}_f = \bar{\boldsymbol{C}} - \bar{\boldsymbol{C}}_d \tag{8b}$$

$$\bar{\boldsymbol{K}} = \bar{\boldsymbol{K}}_d + \bar{\boldsymbol{K}}_f \tag{8c}$$

$$\overline{\boldsymbol{K}}_d = \begin{bmatrix} \overline{\boldsymbol{K}}_s & 0 \\ 0 & \overline{\boldsymbol{K}}_g \end{bmatrix} + \mathrm{diag}[\overline{\boldsymbol{K}}_{sg}], \quad \overline{\boldsymbol{K}}_f = \overline{\boldsymbol{K}} - \overline{\boldsymbol{K}}_d \tag{8d}$$

in which diag[ ] denotes the diagonal entries of matrices. Matrices $\overline{\boldsymbol{C}}_d$ and $\overline{\boldsymbol{C}}_f$ are the diagonal and off-diagonal elements of $\overline{\boldsymbol{C}}$ respectively. Matrices $\overline{\boldsymbol{K}}_d$ and $\overline{\boldsymbol{K}}_f$ are the diagonal and off-diagonal elements of $\overline{\boldsymbol{K}}$ respectively. Thus, the equation (6) can be reconstructed as:

$$\boldsymbol{I}\ddot{\boldsymbol{q}} + \overline{\boldsymbol{C}}_d\dot{\boldsymbol{q}} + \overline{\boldsymbol{K}}_d\boldsymbol{q} = \overline{\boldsymbol{P}} - \overline{\boldsymbol{C}}_f\dot{\boldsymbol{q}} - \overline{\boldsymbol{K}}_f\boldsymbol{q} \tag{9}$$

2. Iterative sequence for stochastic analysis

In the stochastic analysis of combined soil-structure system described in equation (1), the external force vector $\boldsymbol{P}$ is considered to be a series of zero mean Gaussian processes, and the power spectral density (PSD) function vector is denoted as $\boldsymbol{S_P}$.Because of the computational complexity of traditional random vibration theory, the pseudo excitation method is introduced here, and the external force is assumed to be the pseudo harmonic excitation:

$$\overline{\boldsymbol{P}} = \boldsymbol{\Phi}^T\boldsymbol{P}, \quad \boldsymbol{P} = \sqrt{\boldsymbol{S_P}}e^{r\omega t}, \quad r = \sqrt{-1} \tag{10}$$

Define:

$$\boldsymbol{H}_d = (-\boldsymbol{I}\omega^2 + r\omega\overline{\boldsymbol{C}}_d + \overline{\boldsymbol{K}}_d)^{-1} \tag{11}$$

According to equations (9), (10) and (11), the iterative solution of pseudo response $\boldsymbol{q}$ can be given by:

$$\boldsymbol{q}^{(k)} = \boldsymbol{q}_\omega^{(k)}e^{r\omega t} \tag{12a}$$

$$\boldsymbol{q}_\omega^{(k)} = \boldsymbol{H}_d\left[\boldsymbol{\Phi}^T\sqrt{\boldsymbol{S_P}} - (r\omega\overline{\boldsymbol{C}}_f + \overline{\boldsymbol{K}}_f)\boldsymbol{q}_\omega^{(k-1)}\right], \quad k = 1,2,\ldots \tag{12b}$$

in which $\boldsymbol{q}^{(0)} = 0$ and $\boldsymbol{q}_\omega^{(0)} = 0$ are assumed to be the initial pseudo response. If the iterative sequence converges, the exact results of pseudo response $\boldsymbol{q}$ can be obtained by equation (12). Then, the displacement PSD of the soil-structure system can be obtained by:

$$\boldsymbol{S}_U = \boldsymbol{U}^*\boldsymbol{U}^T \tag{13}$$

in which, the superscript * indicates the complex conjugate. $\boldsymbol{U}$ can be obtained by equation (5). Before closing this section, it is useful to generalize equation (13) for response quantities other than nodal displacement. It is well known that a displacement-related response quantity $z(t)$ such as an internal force or stress, can be expressed in terms of the vector of nodal relative displacement, $\boldsymbol{U}$

$$z(t) = \boldsymbol{\Gamma U} \tag{14}$$

where, $\boldsymbol{\Gamma}$ is an vector of constants. For the internal force in a member, for example, $\boldsymbol{\Gamma}$ is given in terms of the elements of the stiffness matrix of the member. Thus, the PSD of the response quantity $z(t)$ can be expressed as:

$$S_z = (\boldsymbol{\Gamma U})^* (\boldsymbol{\Gamma U})^T = \boldsymbol{\Gamma U}^* \boldsymbol{U}^T \boldsymbol{\Gamma}^T = \boldsymbol{\Gamma} S_U \boldsymbol{\Gamma}^T \tag{15}$$

Then, the spectral moments of $\boldsymbol{S}_z$ are defined as follows:

$$\beta_n = \int_{-\infty}^{\infty} \omega^n \boldsymbol{S}_z d\omega \tag{16}$$

3. Convergence condition

Let $\boldsymbol{q}^e$, $\boldsymbol{q}_\omega^e$ be the exact value of pseudo response of equation (9) under the pseudo excitation, and $\boldsymbol{q}^{(k)}$, $\boldsymbol{q}_\omega^{(k)}$ be the approximate value corresponding to the kth step of the iterative process. Then, equation (12) for exact pseudo response can be rewritten as:

$$\boldsymbol{q}_\omega^e = \boldsymbol{H}_d \left[\boldsymbol{\Phi}^T \sqrt{\boldsymbol{S_P}} - \left(r\omega \tilde{\boldsymbol{C}}_f + \tilde{\boldsymbol{K}}_f \right) \boldsymbol{q}_\omega^e \right] \tag{17}$$

Subtracting equation (12b) from equation (17), and noting that the error of kth iterative step can expressed as $\delta^{(k)} = \boldsymbol{q}^e - \boldsymbol{q}^{(k)}$, $\delta_\omega^{(k)} = \boldsymbol{q}_\omega^e - \boldsymbol{q}_\omega^{(k)}$, one can obtain:

$$\boldsymbol{\delta}_\omega^{(k)} = -\boldsymbol{H}_d \left(r\omega \tilde{\boldsymbol{C}}_f + \tilde{\boldsymbol{K}}_f \right) \boldsymbol{\delta}_\omega^{(k-1)} \tag{18}$$

Define:

$$\boldsymbol{N}_\omega = -\boldsymbol{H}_d \left(r\omega \tilde{\boldsymbol{C}}_f + \tilde{\boldsymbol{K}}_f \right) \tag{19}$$

This then yields the recursion of the error:

$$\boldsymbol{\delta}_\omega^{(k)} = \boldsymbol{N}_\omega^k \boldsymbol{\delta}_\omega^{(0)} \tag{20}$$

It can be easily known that if $\boldsymbol{N}_\omega^k \to 0$ while $k \to \infty$, then $\delta_\omega^{(k)} \to 0$, which means that the iterative solution of equation (15) converges to zero. This condition is equivalent to the spectral radius ρ_{N_ω} of $\boldsymbol{N}_\omega$, being less that unity for all $\omega \in \mathrm{R}$, that is:

$$\rho_{N_\omega} \leq 1 \tag{21}$$

While the condition described in equation (21) is satisfied, the stochastic response obtained by equation (12) will converge to the exact results. Moreover, in order to evaluate the convergence speed of the iteration, we define the error as follows:

$$\boldsymbol{Err}_k(i) = \underset{\omega\in R}{\mathrm{RMS}}\left(\left|\boldsymbol{q}_\omega^e(i) - \boldsymbol{q}_\omega^{(k)}(i)\right|\right) \Big/ \underset{\omega\in R}{\mathrm{RMS}}\left(\left|\boldsymbol{q}_\omega^e(i)\right|\right) \tag{22a}$$

$$v_k = \left(\left\|\boldsymbol{Err}_k\right\| / \left\|\boldsymbol{Err}_0\right\|\right)^{1/k} \tag{22b}$$

where, $\boldsymbol{Err}_k(i)$ is defined as the normalized root mean square (RMS) of norm error of the ith component in kth iteration. v_k is the average reduction factor per iteration for the successive error norms. $\left\|\boldsymbol{Err}_k\right\|$ is the Euclidean norm of the error vector at kth iteration and $\left\|\boldsymbol{Err}_0\right\|$ is the Euclidean norm of initial error vector. Also, we can define:

$$CR_k = -\frac{1}{k}\log\left(\left\|\boldsymbol{Err}_k\right\| / \left\|\boldsymbol{Err}_0\right\|\right) \tag{23}$$

as the average rate of convergence over kth iteration.

NUMERICAL EXAMPLES

The stochastic response of combined system, such as the soil-structure system and primary-secondary system and so on, has been analyzed by using complex eigenvalues and eigenvectoers (Igusa, et al., 1984) and by using the pseudo-force approach but for real eigenvalues and eigenvectoers of the undamped combined system (Jangid and Datta, 1993). Accordingly, the iterative equation has also been improved by adopting the classical modal decomposition (Zavoni et al., 2006). But it can be noted that all the methods are based on the conventional random theory which is usually computational inefficient for stochastic analysis of large systems. And a fast stochastic analysis method which is denoted as the pseudo-excitation method has been developed (Lin, 1992). Therefore, based on the pseudo-excitation method, some stochastic analysis methods for the combined system with non-proportional damping characteristics were proposed (Xu and Zhang, 2001; Lin and Zhang, 2004). However, all of these methods require either the solution of complex eigenvalues problem or the inverse operation of matrices. By overcoming these difficulties in computation, a new iterative method is proposed in this paper with more advantages. The chief advantage is its accuracy in the stochastic analysis of soil-structure interaction system. Moreover, it also provides economic advantages both in terms of the fast stochastic analysis by introducing the pseudo-excitation method and avoiding the determination of eigenvalues and eigenvectors of the damped or undamped combined system by using the classical modal decomposition. To demonstrate the improvement of the method, two models representing the soil-structure interaction system are analyzed in the following section. Numerical comparison with the exact results obtained by the Lin's method (Lin and Zhang, 2004) and the approximate results by forced decoupling method are given to examine the accuracy and convergence of the new method.

1. Simple 2 DOF model

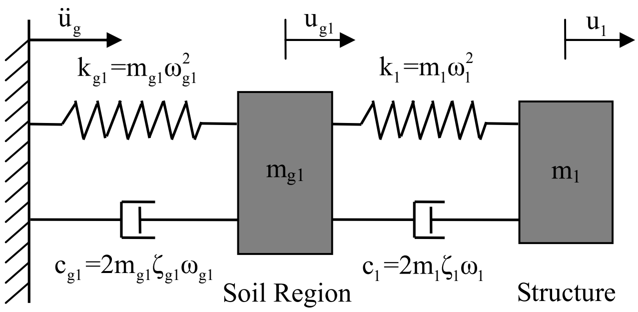

Figure 2: 2 DOF model of soil-structure interaction system

Figure 2 shows a simple 2 degree-of-freedom (DOF) model of soil-structure interaction system. According to the iterative equation proposed in this paper, the displacement PSD of the combined system can be obtained. For the model described in figure 2, the mass, stiffness, damping matrices and displacement response vector of the combined system can be defined as:

$$\boldsymbol{M}=\begin{bmatrix} m_1 & 0 \\ 0 & m_{g1} \end{bmatrix} \tag{24a}$$

$$\boldsymbol{C}=\begin{bmatrix} 2m_1\zeta_1\omega_1 & 0 \\ 0 & 2m_{g1}\zeta_{g1}\omega_{g1} \end{bmatrix}+\begin{bmatrix} 0 & -2m_1\zeta_1\omega_1 \\ -2m_1\zeta_1\omega_1 & 2m_1\zeta_1\omega_1 \end{bmatrix} \tag{24b}$$

$$\boldsymbol{K}=\begin{bmatrix} m_1\omega_1^2 & 0 \\ 0 & m_{g1}\omega_{g1}^2 \end{bmatrix}+\begin{bmatrix} 0 & -m_1\omega_1^2 \\ -m_1\omega_1^2 & m_1\omega_1^2 \end{bmatrix} \tag{24c}$$

$$\boldsymbol{U}=\begin{bmatrix} u_1 \\ u_{g1} \end{bmatrix} \tag{24d}$$

Thus, the linear transformation matrix can be constructed as:

$$\boldsymbol{\Phi}=\begin{bmatrix} \boldsymbol{\Phi}_s & 0 \\ 0 & \boldsymbol{\Phi}_g \end{bmatrix}=\begin{bmatrix} 1/\sqrt{m_1} & 0 \\ 0 & 1/\sqrt{m_{g1}} \end{bmatrix} \tag{25}$$

Then, equation (8) can be written as：

$$\bar{\boldsymbol{C}} = \bar{\boldsymbol{C}}_d + \bar{\boldsymbol{C}}_f = \begin{bmatrix} 2\zeta_1\omega_1 & 0 \\ 0 & 2\zeta_{g1}\omega_{g1} + +2m_1\zeta_1\omega_1/m_{g1} \end{bmatrix} + \begin{bmatrix} 0 & \dfrac{-2m_1\zeta_1\omega_1}{\sqrt{m_1}\sqrt{m_{g1}}} \\ \dfrac{-2m_1\zeta_1\omega_1}{\sqrt{m_1}\sqrt{m_{g1}}} & 0 \end{bmatrix} \tag{26a}$$

$$\bar{\boldsymbol{K}} = \bar{\boldsymbol{K}}_d + \bar{\boldsymbol{K}}_f = \begin{bmatrix} \omega_1^2 & 0 \\ 0 & \omega_{g1}^2 + \omega_1^2 m_1/m_{g1} \end{bmatrix} + \begin{bmatrix} 0 & \dfrac{-m_1\omega_1^2}{\sqrt{m_1}\sqrt{m_{g1}}} \\ \dfrac{-m_1\,\omega_1^2}{\sqrt{m_1}\sqrt{m_{g1}}} & 0 \end{bmatrix} \tag{26b}$$

Upon substitution of equation (26) into equation (11) one can obtain:

$$\begin{aligned} \boldsymbol{H}_d &= (-\boldsymbol{I}\omega^2 + r\omega\bar{\boldsymbol{C}}_d + \bar{\boldsymbol{K}}_d)^{-1} \\ &= \begin{bmatrix} \omega_1^2 - \omega^2 + r\omega(2\zeta_1\omega_1) & 0 \\ 0 & \left(\omega_{g1}^2 + m_1\omega_1^2/m_{g1}\right) - \omega^2 + r\omega\left(2\zeta_{g1}\omega_{g1} + 2m_1\zeta_1\omega_1/m_{g1}\right) \end{bmatrix}^{-1} \end{aligned} \tag{27}$$

For seismic analysis of the combined system, the external force can be expressed as：

$$\boldsymbol{P} = -\boldsymbol{M}\begin{bmatrix}1\\1\end{bmatrix}\ddot{u}_g \tag{28}$$

Assume that the acceleration PSD of the ground motion $\ddot{u}_g$ is $S_{\ddot{u}_g}$, the pseudo excitation in equation (10) is given by:

$$\boldsymbol{P} = -\boldsymbol{M}\begin{bmatrix}1\\1\end{bmatrix}\sqrt{S_{\ddot{u}_g}}e^{r\omega t}, \quad r = \sqrt{-1} \tag{29}$$

The stochastic model of ground motion, the Kanai-Tajimi model, is adopted here, which can be described as:

$$S_{\ddot{u}_g} = \frac{\omega_g^4 + (2\zeta_g\omega_g\omega)^2}{(\omega_g^2 - \omega^2)^2 + (2\zeta_g\omega_g\omega)^2}S_0 \tag{30}$$

in which, $\omega_g = 13.96\text{rad/s}$ and $\zeta_g = 0.72$, which correspond to the site type 1 and the earthquake classification 3 defined in Chinese code. The intensity is assumed to be $S_0 = 0.06\,\text{m}^2/\text{s}^3$. Suppose that $m_1 = 8\times10^6\,\text{kg}$, $m_{g1} = 8\times10^8\,\text{kg}$, $\zeta_1 = 0.05$, $\zeta_{g1} = 0.1$, $\omega_1 = 10\text{rad/s}$, $\omega_{g1} = 15\text{rad/s}$. Thus, the pseudo response can be calculated by the iterative equation (12), and accordingly the PSD and spectral moments can also be obtained by equations (5), (13), (15) and (16).

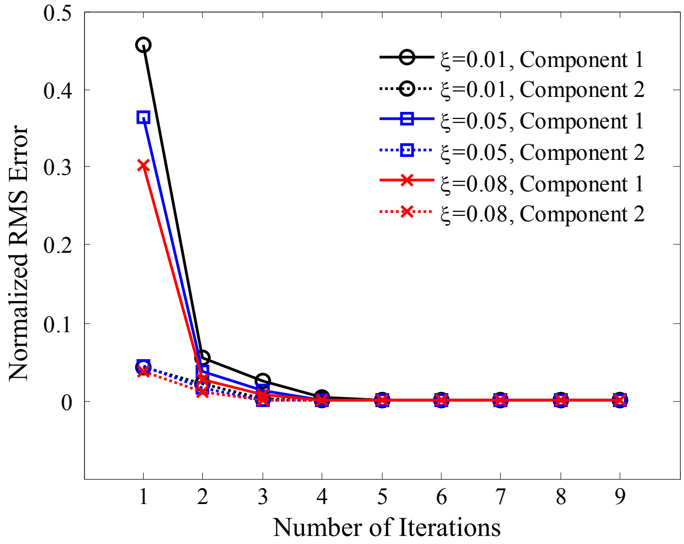

Figure 3: Normalized RMS error of modal coordinate components versus number of iterations

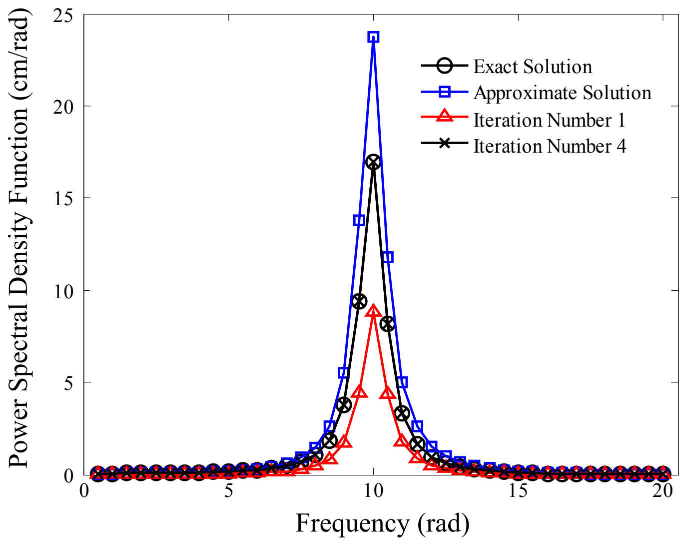

Figure 4: Power spectral density function of structural relative displacement with damping ratio of soil region ξ =0.2

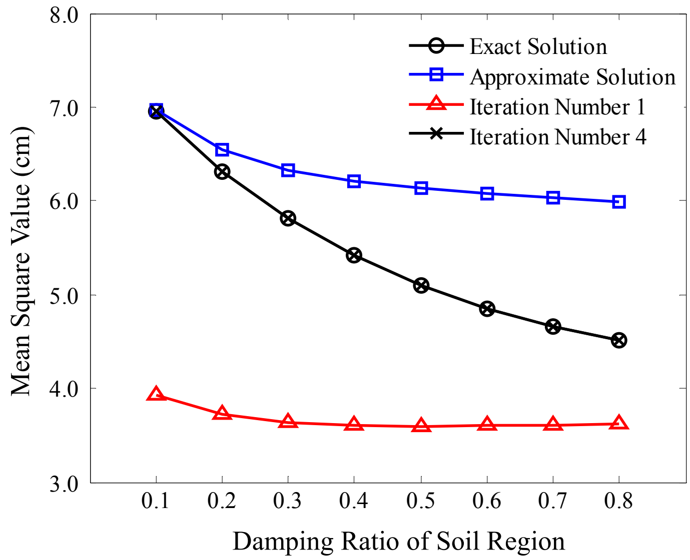

Figure 5: Mean square value of structural relative displacement versus damping ratio of soil region

Figure 3 illustrates the convergence speed of the modal coordinate components by studying the variety of $\boldsymbol{Err}_k(i)$, which represents the normalized root mean square (RMS) of norm error of the ith component in kth iteration. It is noted from figure 3 that $\boldsymbol{Err}_k(i)$ decreases dramatically along with iterative times increasing which indicates the availability of the iterative method. Moreover, in the simple 2 DOF model, the structural relative displacement with respect to the soil region is concerned, and it can be obtained by equations (14) by setting $\boldsymbol{\Gamma} = [1 \quad -1]$. Accordingly, in order to examine the accuracy of the iterative method, the PSD and mean square values of structural relative displacement corresponding to the exact solution, approximate solution and iterative solution are given in figure 4 and figure 5, in which the exact solution is obtained by Lin's method (Lin and Zhang, 2004) and the approximate solution by the forced decoupling method. Figure 5 shows that the forced decoupling method might introduce gross obvious error for high non-proportional damping, and the iterative method can produce exact solution after a few iterations. Thus, the proposed method can be used for the stochastic analysis of the simple 2 DOF model of soil-structure interaction system.

2. Lumped parameters model

In order to study the validity of the proposed iterative method for stochastic analysis of soil-structure interaction system, a more applicable model compared with simple 2 DOF model, which is called lumped parameters model, would be utilized in this section. In the last three decades, several lumped parameters models for soil-structure system have been proposed (Wolf and Somaini, 1983; De Barros and Luco, 1990; Jean, et al., 1990; Ruan and Lin, 1996), which commonly consist of an assembly of mass, damping and stiffness elements independent of frequencies. Because the lumped parameters model can account for the seismic characteristics of practical soil-structure interaction system in usual cases, it can be used to simulate the combined soil-structure system. In this section, Ruan and Lin's 2 DOF lumped parameters model are adopted. Figure 6 is the schematic diagram of the model of combined soil-structure system.

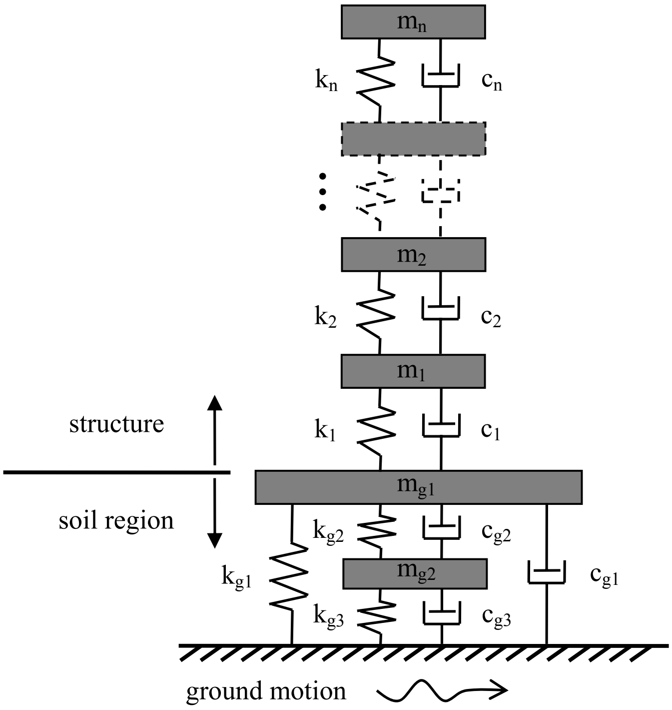

Figure 6: 2 DOF lumped parameters model of soil-structure interaction system

For the 2 DOF lumped parameters model of soil-structure interaction system described in Figure 6, the mass, stiffness and damping matrices are defined as:

$$\boldsymbol{M} = \begin{bmatrix} \boldsymbol{M}_s & 0_{n\times 2} \\ 0_{2\times n} & \boldsymbol{M}_g \end{bmatrix}, \quad \boldsymbol{M}_s = \begin{bmatrix} m_n & & 0 \\ & \ddots & \\ 0 & & m_1 \end{bmatrix}, \quad \boldsymbol{M}_g = \begin{bmatrix} m_{g1} & 0 \\ 0 & m_{g2} \end{bmatrix} \tag{31a-c}$$

$$\boldsymbol{K} = \begin{bmatrix} \boldsymbol{K}_s & 0 \\ 0 & \boldsymbol{K}_g \end{bmatrix} + \boldsymbol{K}_{sg}\,, \quad \boldsymbol{K}_s = \begin{bmatrix} k_n & -k_n & & 0 \\ -k_n & k_n + k_{n-1} & -k_{n-1} & \\ & & \ddots & \\ 0 & & -k_2 & k_2 + k_1 \end{bmatrix} \tag{32a-b}$$

$$\boldsymbol{K}_g = \begin{bmatrix} k_{g1}+k_{g2} & -k_{g2} \\ -k_{g2} & k_{g2}+k_{g3} \end{bmatrix}, \quad \boldsymbol{K}_{sg} = \begin{bmatrix} 0_{(n-1)\times(n-1)} & 0_{(n-1)\times 1} & 0_{(n-1)\times 1} & 0_{(n-1)\times 1} \\ 0_{1\times(n-1)} & 0 & -k_1 & 0 \\ 0_{1\times(n-1)} & -k_1 & k_1 & 0 \\ 0_{1\times(n-1)} & 0 & 0 & 0 \end{bmatrix} \tag{32c-d}$$

$$\boldsymbol{C} = \begin{bmatrix} \boldsymbol{C}_s & 0 \\ 0 & \boldsymbol{C}_g \end{bmatrix} + \boldsymbol{C}_{sg}, \quad \boldsymbol{C}_s = \alpha \boldsymbol{M} + \beta \boldsymbol{K} \tag{33a-b}$$

$$\boldsymbol{C}_g == \begin{bmatrix} c_{g1}+c_{g2} & -c_{g2} \\ -c_{g2} & c_{g2}+c_{g3} \end{bmatrix}, \quad \boldsymbol{C}_{sg} = \begin{bmatrix} 0_{(n-1)\times(n-1)} & 0_{(n-1)\times 1} & 0_{(n-1)\times 1} & 0_{(n-1)\times 1} \\ 0_{1\times(n-1)} & 0 & -c_1 & 0 \\ 0_{1\times(n-1)} & -c_1 & c_1 & 0 \\ 0_{1\times(n-1)} & 0 & 0 & 0 \end{bmatrix} \tag{33c-d}$$

Assume that the PSD of the ground motion $\ddot{u}_g$ is $S_{\ddot{u}_g}$, the pseudo excitation in equation (10) is given by:

$$\boldsymbol{P} = -\boldsymbol{M} 1_{(n+2)\times 1} \sqrt{S_{\ddot{u}_g}} e^{r\omega t}, \quad r = \sqrt{-1} \tag{34}$$

In the numerical example, a 10-storey shear structure is selected with all the storey stiffnesses equal to 2×10^8 N/m and all the floor masses equal to 8×10^5 Kg. The Rayleigh damping is adopted, and a 5% damping ratio is considered for 1st and 5th mode of the structure. Only the horizontal motion of the combined system is considered in the example. Besides, the soil-structure interaction system is subjected to a white noise process with spectral amplitude equal to $0.6 m^2/s^3$. For the simplified model of soil-structure interaction system, the 8 parameters of 2-DOF lumped parameters model of soil region are given in the literatures (Ruan and Lin, 1996; Wang et al., 2007):

$$\begin{cases} m_{g1} = m_{fc} m_{f1} \\ m_{g2} = m_{fc} m_{f2} \end{cases}, \quad \begin{cases} c_{g1} = c_{fc} c_{f1} \\ c_{g2} = c_{fc} c_{f2} \\ c_{g3} = c_{fc} c_{f3} \end{cases}, \quad \begin{cases} k_{g1} = k_{fc} k_{f1} \\ k_{g2} = k_{fc} k_{f2} \\ k_{g3} = k_{fc} k_{f3} \end{cases} \tag{35a-c}$$

in which m_{fc}, c_{fc} and k_{fc} are the normalization factors. For strip foundation, the parameters' values can be given by:

$$k_{fc} = \pi \rho v_s^2, \quad m_{fc} = k_{fc} \left(r_0 / v_s \right)^2, \quad c_{fc} = k_{fc} \left(r_0 / v_s \right) \tag{36a-c}$$

in which $\rho = 2.6\times10^3 kg/m^3$ denotes the soil density. $v_s = 250 m/s$ denotes shear wave velocity. $r_0 = 50m$ is the width of the strip foundation. In this example, the poisson ratio is assumd to be 0.33, then the values of normalization factors can be obtained:

$$m_{f1} = 0.0013, \quad m_{f2} = 0.347 \tag{37a-b}$$

$$k_{f1} = 2.085, \quad k_{f2} = -0.447, \quad k_{f3} = 0.761 \tag{37c-e}$$

$$c_{f1} = 1.149, \quad c_{f2} = -0.509, \quad c_{f3} = 1.194 \tag{37f-h}$$

In order to construct equation (12), the linear transformation matrix is obtained by equations (3) and (4). As same as example 1, the Kanai-Tajimi model is adopted as the stochastic model of ground motion, and the pseudo response of the combined soil-structure system can be calculated by an iterative process, and accordingly the PSD and spectral moments can be obtained by equations (5), (13), (15) and (16). Figure 7 and figure 8 illustrate the convergence speed of the iterative method. The relative displacement of 1st, 5th, 10th floor with respect to the soil region is selected as the research objects, and they can be obtained by equations (14) by setting $\boldsymbol{\Gamma}_1 = [1 \quad 0_{1\times9} \quad -1 \quad 0]$, $\boldsymbol{\Gamma}_5 = [0_{1\times4} \quad 1 \quad 0_{1\times5} \quad -1 \quad 0]$ and $\boldsymbol{\Gamma}_{10} = [0_{1\times9} \quad 1 \quad -1 \quad 0]$. Accordingly, the mean square values of displacement of the 1st, 5th and 10th floor corresponding to the first several iterative times are shown in figure 9. For comparison, the exact results given by Lin's method and the approximate results given by the forced decoupling method are also shown in the figure 9. It can be seen that the results obtained by the iterative method is close to the exact results after a few iterations.

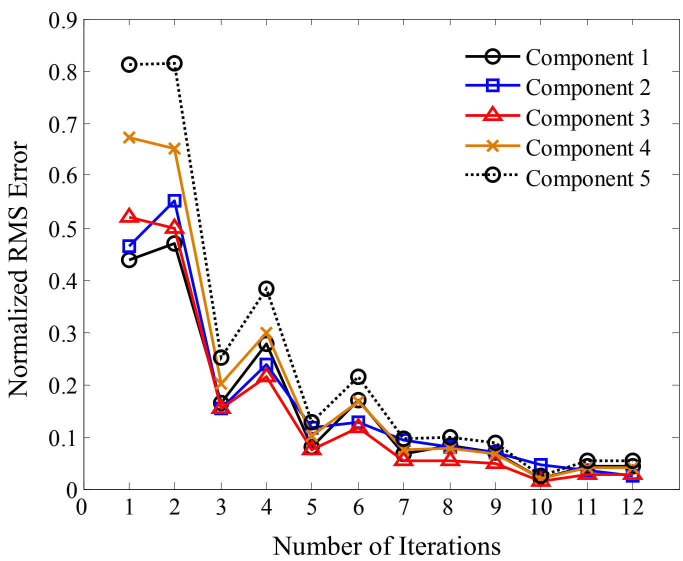

Figure 7: Normalized RMS error of first 5 modal coordinate components versus number of iterations

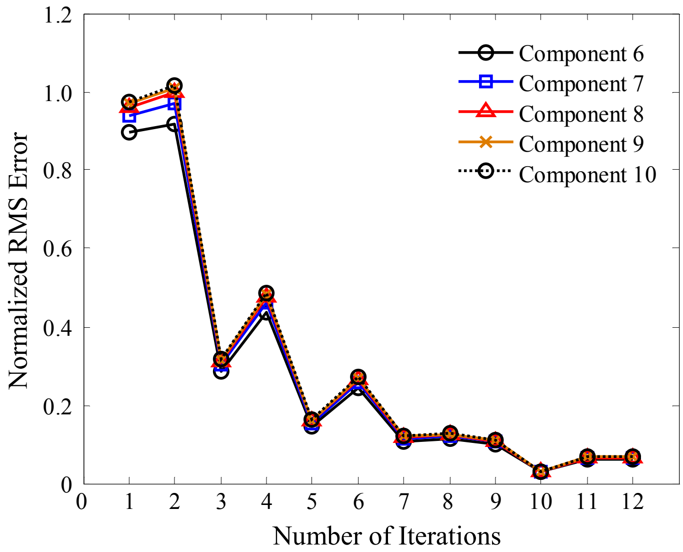

Figure 8: Normalized RMS error of latter 5 modal coordinate components versus number of iterations

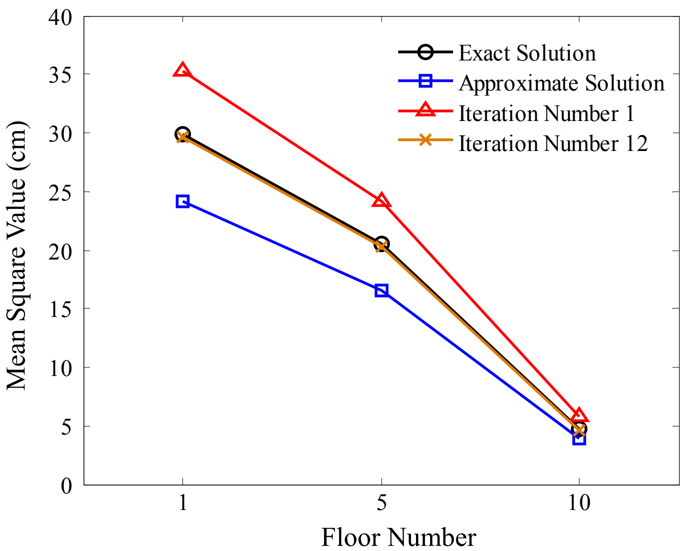

Figure 9: Mean square value of relative displacement versus of floor 1, 5 and 10 with respect to the soil region

CONCLUSIONS

In this paper, a fast stochastic analysis method of soil-structure interaction system has been presented. In the new method, the pseudo-excitation method and the classical modal decomposition is adopted to derivate an iterative sequence, which can exactly account for the non-proportional damping and dynamic interaction between the soil region and the structure without obviously increasing computational effort. Furthermore, the new method's accuracy and convergence have been proved by some numerical examples belonging to two different models of soil-structure interaction system.

ACKNOWLEDGEMENTS

The authors would like to thank the project (No. IRT0518) supported by the program for changjiang scholars and innovative research team in university, and the project (No. B08014) supported by the program of introducing talents of discipline to universities.

REFERENCES

1. Seed, H.B., Hwang, R.N., Lysmer, J. (1975) "Soil-Structure Interaction Analyses for Seismic Response," Journal of the Geotechnical Engineering Division, 101(5): 439-457.
2. Veletsos, A.S., Prasad, A.M. (1989) "Seismic interaction of structures and soils: stochastic approach," Journal of Structural Engineering, 115(4): 935-956.
3. Zhang, X., Wegner, J.L., Haddow, J.B. (1999) "Three-dimensional dynamic soil-structure interaction analysis in the time domain," Earthquake Engineering and Structural Dynamics, 28(12): 1501-1524.
4. Lou, M.L., Wu, J.N. (1999) "Seismic response analysis of pile foundation structure system," China Civil Engineering Journal, 32(5): 56-61. (in Chinese)
5. Ghiocel, D.M., Ghanem, R.G. (2002) "Stochastic finite-element analysis of seismic soil-structure interaction," Journal of Engineering Mechanics, 128(1): 66-77.
6. Gao, Q., Lin, J.H., Zhong, W.X., Howson, W.P., and Williams, F.W. (2009) "Isotropic layered soil–structure interaction caused by stationary random excitations," International Journal of Solids and Structures, 46(3-4): 455-463.
7. Elishakaff, I., Lyon, H.R. (1986) "Random vibration-status recent developments," Elsevier Science Publishers, New York.
8. Foss, K.A. (1958) "Coordinates which uncouple the equation of motion of damped linear dynamic systems," Journal of Applied Mechanics, 25(1): 361-364.
9. Lou, M.L., Duan, Q., Chen, G.D. (2003) "Modal perturbation method and its applications in structural systems," Journal of Engineering Mechanics, 169(8): 935-943.

10. Karen, K., Mohsen, G.A. (2005) "New approaches for non-classically damped system eigenanalysis," Earthquake Engineering and Structural Dynamics, 34(9): 1073-1087.
11. Fernando, C., María, J.E. (2006) "Computational methods for complex eigenproblem in finite element analysis of structural systems with viscoelastic damping treatments," Computer Methods in Applied Mechanics and Engineering, 195(44-47): 6448-6462.
12. Hea, J.J., Jianga, J.S., B. Xu, B., (2007) "Modal reanalysis methods for structural large topological modifications with added degrees of freedom and non-classical damping," Finite Elements in Analysis and Design, 44(1-2): 75-85.
13. Claret, A.M., Venancio, F.F. (1991) "A modal superposition pseudo-force method for dynamic analysis of structural systems with non-proportional damping," Earthquake Engineering and Structural Dynamics, 20(4): 303-315.
14. Lin, F.B., Wang, Y.K., Cho, Y.S. (2003) "A pseudo-force iterative method with separate scale factors for dynamic analysis of structures with non-proportional damping," Earthquake Engineering and Structural Dynamics, 32(2): 329-337.
15. Jandid, R.S., Datta, T.K. (1993) "Spectral analysis of systems with non-classical damping using classical mode superposition technique," Earthquake Engineering and Structural Dynamics, 22(8): 723-735.
16. Zavoni, E.H., Pérez, A.P., and Cicilia, F.B. (2006) "A method for the transfer function matrix of combined primary-secondary systems using classical modal decomposition," Earthquake Engineering and Structural Dynamics, 35(2): 251-266.
17. Lou, M.L., Zhao, Y.L. (1994) "The substructure method for dam-foundation dynamic interaction analysis," Journal of Vibration Engineering, 7(2): 161-166. (in Chinese)
18. Jin, F., Zhang, C.H., Wang G.L. (1997) "A time domain model of arch dam-rock foundation interaction," China Civil Engineering Journal, 30(1): 43-51. (in Chinese)
19. Bi, J.H., Zhang, H. (2003) "Anti-seismic study on metrotunnel based on coupling numerical analysis method," Rock and Soil Mechanics, 24(5): 800-803. (in Chinese)
20. Li, J.B., Chen, J.Y., Lin, G. (2004) "Finite element damping-extraction method for dynamic interaction time domain analysis of non-homogeneous unbounded rock" Chinese Journal of Geotechnical Engineering, 26(2): 263-267.
21. Lin, J.H. (1992) "A fast CQC algorithm of PSD matrices for random seismic responses," Computers and Structures, 44(3): 683-687.
22. Xu, Y.L., Zhang, W.S. (2001) "Modal analysis and seismic response of steel Frames with Connection Dampers," Engineering Structures, 23(4): 385-396.
23. Lin, J.H., Zhang, Y.H. (2005) "Vibration and Shock Handbook, Chapter 30: Seismic Random Vibration of Long-span Structures," CRC Press: Boca Raton, FL.
24. Igusa, T., Kiureghian, D.A., Sackman, J. (1984) "Modal decomposition method for stationary response of non-classically damped systems," Earthquake Engineering and Structural Dynamics, 12(1):121-136.
25. Jangid, R.S., Datta, T.K. (1993) "Spectral analysis of systems with non-classical damping using classical mode superposition technique," Earthquake Engineering and Structural Dynamics, 22(8):723-735.
26. Wolf, J.P., Somaini, D.R. (1983) "Approximate dynamic model of embedded foundation in time domain," Earthquake Engineering and Structural Dynamics, 14(5): 683-703.
27. De Barros, F.C.P., Luco, J.E. (1990) "Discrete models for vertical vibrations of surface and embedded foundation," Earthquake Engineering and Structural Dynamics, 19(2): 289-303.

28. Jean, W.Y., Lin, T.W., Penzien, J. (1990) “System parameter of soil foundation for time domain dynamic analysis,” Earthquake Engineering and Structural Dynamics, 19(4): 541-553.
29. Ruan, M.T., Lin, G. (1996) “2-DOF lumped-parameter model of dynamic impedances of foundation soils,” Journal of Dalian University of Technology, 36(4):477-482. (in Chinese)
30. Wang, M.S., Pan, G.D., Zhou, X.Y. (2007) “Soil-structure interaction analysis based on the soil lumped parameters model,” Journal of University of Science and Technology Beijing, 29(1): 5-10. (in Chinese)